

## **Mathematics problem solving**

**What factors inhibit student achievement, what factors are effective in raising achievement?**

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### **Acknowledgements**

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My sincere appreciation and thanks to the principals, teachers and curriculum leaders, who gave their time so generously, and engaged in open and frank discussions sharing their professional opinions and experiences to support my research.

Thank you to the students who provided such clear insights into their experiences of mathematics and what works and doesn't, to help them to solve problems.

### **Executive Summary**

Many factors impact on children's ability to solve problems in mathematics. The outcomes of this research are very consistent with wider research. Teachers, students and Principals have all identified significant factors that contribute to children's success in problem solving, and to factors that inhibit that success.

All schools involved in the project have strategies in place to further develop children's ability to solve problems, and they are proactive in planning and implementing professional development to support and further develop teacher professional learning and capability.

Five of the schools involved in this research are using teaching as inquiry to focus on practices that contribute to student achievement in mathematics.

The inclusion of student voice in this project is intended to provide relevant, perceptive, insightful, and at times, humorous messages for teachers and school leaders to consider. The children clearly seek to learn, to understand the relevance of mathematics to their world, and to develop their mathematical knowledge.

## **Purpose**

To determine:- What are the factors that inhibit student achievement in mathematics problem solving? What teaching and learning strategies/factors are effective in raising achievement?

## **Rationale**

To carry out qualitative research, to gain insights from students, teachers and Principals to identify common themes, and potential solutions, to support students success in mathematics problem solving.

## **Background**

“Mathematics is the most international of all curriculum subjects, and mathematical understanding influences decision making in all areas of life, private, social and civil. (Effective Pedagogy in Mathematics P6)

Mathematics problem solving is an area of achievement that is critical for our students' future success. The ability to solve problems in mathematics in authentic and meaningful contexts, is a key focus and specific expectation, for children to achieve the National Standards in mathematics.

“While knowledge is critically important for mathematical understanding, its primary role is to facilitate the student's solving of problems and modelling of situations”. (New Zealand Mathematics Standards P 10)

This research includes student voice to provide insights from student perspective that may further enable teachers and school leaders to “tailor education to the learner.” (Ka Hikitia)

## **Methodology**

- Undertaking relevant professional readings
- Interviewing Principals, and where appropriate school mathematics leaders
- Implementing student surveys
- implementing teacher surveys (survey monkey)
- Carrying out class problem solving activities with students, and engaging student voice

## Findings

Principals from seven schools, (four full Y1-8 primary schools, two Y1-6 primary schools and one Y7-13 school ) were interviewed for this research project, using a questionnaire consisting of eight questions designed to gather information in a structured way around the central purpose of the project. Not all Principals answered all questions, and in several cases Principals provided multiple responses to some questions.

The summarised trends and outcomes of these interviews provide valuable insights into the depth and scope of activities that schools are engaged in to support and develop children's problem solving, and to strengthen teachers' learning and capability.

### Principals' Perspectives

#### **Question 1 How well do you consider children in your school are able to solve mathematics problems?**

- Most principals considered that children were demonstrating good to strong achievement in problem solving.
- Schools have pockets of very high achievement, with wide ranges in ability evident in most schools.
- Where problems involve the use of simple numbers and only one or two steps, the children achieve well.
- Where problems are simply stated and do not involve inference, children read and interpret problems well.
- Children who are achieving highly in mathematics problem solving, are likely to have correspondingly high achievement in literacy.
- Maori students are achieving within the usual achievement patterns for all students in the schools.

#### **Question 2 Is this the same across all strands?**

- Students are developing their ability to apply problem solving strategies across strands.
- For many children there is a need for a very strong focus on number, therefore less time is spent on the other strands, and problem solving is not well developed.
- In one school, students demonstrate confidence in solving problems in geometry and statistics.
- 3 schools involved in this research are collecting and analysing data in problem solving across strands. The other schools indicated that this an area for further development and support.

#### **Question 3 Are there any strands or concepts that pose more or less difficulty than others?**

- Geometry
- Fractions at all levels - proportions and ratios at senior level
- Converting decimals to percentages, to fractions
- Algebra

- Using large numbers in all operations
- Understanding place value – particularly with large numbers
- Application of problem solving in practical situations across the curriculum.

#### **Question 4 What factors do you consider are contributing to children's success?**

- Excellent resourcing
- Children using materials in their problem solving
- All classes teaching problem solving
- Whole school teaching of the Numeracy Project.
- Maintaining focus on basic facts knowledge
- Analysis of assessment data to identify and provide support for target children.
- Ability groupings
- Cooperative grouping for problem solving
- Providing safe, supportive, compassionate environments encouraging risk taking, welcoming children's strategies, developing resilience, fostering a "can do" attitude.
- Maori and Pasifika students achieve best when using contexts, concepts and resources that reflect their cultural settings and utilise their prior experience.
- Introducing "Figure It Outs" in the reading programme, Y4 up, prior to using them in the mathematics programme to strengthen understanding and interpretation of problems.
- Staff meeting workshops focused on mathematics eg place value.
- Consistent NUMPA/GLOSS testing across the school.
- Quality teachers and quality teaching – positive teacher attitudes.
- Focussing teaching on specific concepts.
- Involving parent community through information evenings, open mornings, maths week, fun challenges including parents, including maths games each week in the school newsletter, parent helper support in the programme.
- Sharing mathematics leadership – having all levels of the school represented in the Mathematics curriculum team.

#### **Question 5 What factors are inhibiting success?**

- Lack of accurate evaluation of children's prior knowledge of concepts prior to introducing new learning, eg time, measurement.
- Children's negative experience of mathematics impacts on their willingness to take risks, and many children have a negative perception of their overall ability to achieve in mathematics.
- Student disengagement, supported by a sense that it is socially acceptable not to be good at mathematics.
- Aspects of the Numeracy Project such as the perceived narrow focus, the lack of emphasis on problem solving, and the range of strategies – children find this particularly confusing.
- Teacher knowledge and confidence in problem solving.
- Lack of specific teaching of the language of mathematics used in problem solving, and the application of reading skills, particularly inference to enable children to interpret and understand questions.

- Lack of teaching of analytical tools and strategies to support children in developing logical thought and sequential processes.
- Teacher knowledge and capability in integrating mathematics across the curriculum, and in providing authentic problem solving contexts.

### **Question 6 What strategies have teachers used that have raised achievement?**

- Having a strong learner focus, developing student voice through students self-evaluating their learning stage, and sharing their work with an authentic audience on class blogs.
- Having high expectations of students.
- Sharing learning intentions – using modelling books to record concept lessons.
- Having a wide range of materials available for student use at any time in the programme.
- Daily problem solving in mathematics programmes.
- Using assessment to identify individual needs – supporting and responding to these.
- Using ability groups
- Using Website tools and applications eg:- Sum dog Tutpup
- Focussing Teaching as inquiry on target children in mathematics.
- Providing challenges such as Otago Problem solving, Mathswell, ASB challenge, ICAS, Mathletics.

### **Question 7 Have you implemented any school-wide initiatives regarding problem solving?**

- Direct investment – providing budgets to purchase resources
- School funded Mathletics
- In-school support – involving all teachers, developing teacher practice, utilising teacher strengths – skilled teachers modelling concepts for other teachers/coaching supporting other colleagues- teachers released for observing and being observed, and reflection and discussions – identifying next professional practice steps.
- Call back days/staff workshops focussing on conceptual aspects of mathematics,
- Use of teaching as inquiry – teachers carrying out research, identifying aspects of practice to develop, implementing changes and providing evidence of professional development and shifts in practice and achievement.
- School leadership articulating and establishing a common shared direction and purpose, heightening teachers' awareness and accountability for progress and achievement.
- Syndicate focus on mathematics achievement.
- Major teacher development over 2 years, to grow mathematical and pedagogical knowledge and understanding, using external facilitator to develop understanding of teacher performance, and provide individually targeted professional coaching to develop teaching practice.
- Linking mathematics in all strands across the curriculum

- Strengthening teacher analysis, understanding and use of achievement information to support achievement.
- Using the SOLO Taxonomy to develop and evaluate children's levels of understanding in problem solving.
- Participation in ALIM to accelerate progress of children at risk

### **Question 8 How has this impacted on student problem solving and teacher programmes?**

- Children own their own learning – learning is transformed – skills and knowledge are a focus from NE onwards.
- Improvements are evident in all groups including Maori students.
- Teaching programmes closely incorporate the outcomes of assessment.
- Overall mathematics achievement and problem solving has been raised.
- Teacher practice and confidence in their efficacy has increased, and student achievement has been raised

### **Question 9 What challenges is your team currently exploring with problem solving?**

- Mentoring/coaching leaders – developing collegiality, mutual support, professional feedback, principal and team leaders attend team meetings – observe teachers ideas/comments/ interview children to determine support and further development.
- Developing teacher capability – building expertise within the teaching team.
- Moving locus of control – developing children's ability to evaluate their own learning and articulate their next step.
- Developing professional development in problem solving – within the professional cluster.
- Actively recruiting specialist teachers into the management team.
- Developing children's reading ability, particularly the application of their reading skills to interpret and solve mathematics problems.

<b>Teacher Perspectives</b>
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Five mathematics lead teachers were interviewed, and teachers from a range of class levels from Y 0 – 8, completed the following 8 question survey. These summarised outcomes, capture key themes from these activities.

### **Question1:- How well do you consider children in your class are able to solve problems?**

- Within a group with the teacher and /or adult they are quite successful.
- Children are “pretty good” at problem solving as long as they understand the question.
- They have good knowledge of strategies
- Generally very well, a small handful struggle to connect with the operation required to be used.

- They are good at solving problems involving addition and subtraction and multiplication that involve numbers under 100, and that involve only one step.
- Some are more able to understand mathematical vocabulary involved than others.
- Average to below average
- Some are capable to their expected level, but most are below expected level
- There is a wide range of ability.

### **Question 2 What factors do you consider contribute to children's success in problem solving?**

- Being explicitly taught problem solving strategies such as “act it out”, “make a list”, using mathematical knowledge “guess and check”, “draw a diagram” etc.
- Creating an environment where risk taking with learning is encouraged and celebrated – makes it okay for kids to try again if they don't get it right the first time, or to try a range of strategies.
- Integrating problem solving strategies as an ongoing part of classroom language – not just when you are “doing” problem solving.
- Good quality teaching.
- Teacher knowledge of concepts.
- Supporting children's cognitive ability, modelling and teaching them to be able to plan, to be systematic, logical and organised in their mathematical thinking.
- Providing rich relevant mathematical tasks applied to real life situations.
- Regular exposure to a variety of types of problems.
- Being confident in their ability to use a variety of strategies to solve problems.
- Previous successful experience in problem solving builds children's confidence and willingness to persevere to solve problems.
- Focussing on the process not the answer.
- Using resources and materials whenever needed.
- Having a solid base of number knowledge and mathematical thinking
- Being able to read, interpret and understand the problem, having a good understanding of mathematical language.

### **Question 3 What factors do you consider inhibit their success in problem solving?**

- Focusing on the answer not the process.
- Lack of materials and resources, and lack of children's experience in working with concrete materials to help understand and solve problems.
- Perceived value of one strategy over another, rather than confidence and knowledge in using a range of strategies.
- Programmes not providing enough time to allow students to think in depth when solving problems.
- Insufficient sharing/discussion to explore how different people have solved a problem.
- Teachers not teaching through a problem solving approach
- Insufficient teaching and exposure to mathematical language, to enable children to read and understand mathematical language used in problems.
- Previous negative experience of mathematics problem solving.

- Lack of development of key competencies such as managing self and thinking skills, impacts on students' ability to be organised, to concentrate and keep on task, and to know where and how to start the problem solving process.
- Limited number knowledge

**Question 4:- What factors have you used that have been successful in raising their confidence and success?**

- Actively teaching strategies for problem solving.
- Providing rich relevant mathematical tasks that the children can relate to, applied to real life situations.
- Having materials accessible all the time such as, activities, games, competition, ICT.
- Lots of talking through with students about how they have solved problems.
- Breaking down the task so that it is easier to understand and is in achievable chunks.
- Encouraging students to take risks – “finding out where we went wrong so we can learn from it”.
- Questioning students to prompt them towards finding a strategy or solution.
- Clear learning intentions, having clear success criteria that show the necessary steps to complete the task.
- Teacher modelling of new concepts and problem solving strategies, having modelling books available for student reference.
- Positive feedback and feed-forward
- Working with a buddy/learning/coaching – another child becomes the teacher.
- Taking apart the wording of a problem, discussing it and looking for key words to help understand what is required to solve the problem.
- Specific teaching of the stages in problem solving.
- Discussion and working through problems in cooperative groupings.
- Focussing on thinking and working it out.

**Question 5:- Are there any strands or concepts that you have noticed pose greater or lesser difficulty in problem solving?**

- Fractions is a major difficulty for most children
- Other areas of difficulty identified are:-, algebra at senior levels, decimals, proportions, percentages, measurement, place value – tenths, hundredths, thousandths.
- Problems that involve large numbers.
- Children experience difficulty in working in other mathematics strands if their base mathematical knowledge is weak.
- Children experience difficulty if the problems lack relevance and are not age and ability appropriate to them.
- Problems with multiple steps involving logical thinking
- Reading and understanding mathematical language used in the wording of problems.

**Question 6:- For areas that pose difficulty, what reasons have you identified for this difficulty?**

- When faced with a problem involving fractions, many children don't know where to start, and don't like to take risks.
- Teacher knowledge and understanding of both fractions and algebra.
- Lack of previous experience in other strands, not enough teaching, practice, and exposure to other strands

**Question 7 :- My students prefer to solve problems**

The children were presented with four choices of learning situation and were asked to indicate their preferences. The majority of students prefer to work with a buddy, their second preference was to work on their own. A smaller number of students prefer to work in the whole class situation. The least preferred option was to work in a group.

On their own 4 classes

With a buddy 8 classes

In a group 0

Whole class 2 classes

**Question 8 :- Which situations have you noticed they experience greatest success and demonstrate the most confidence?**

The majority of teachers identified that children perform best when working with a buddy, with the second most successful situation being working in a group. One class identified that children experience the most confidence and success when working in the whole class situation. No teachers identified children working alone as a situation that generates success and confidence.

On their own

With a buddy 8 classes

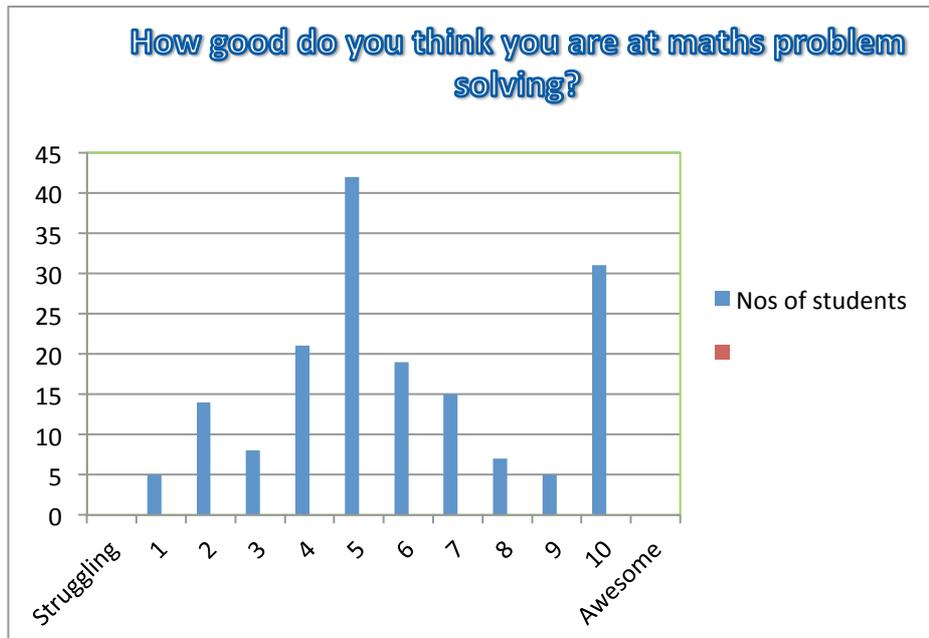
In a group 4 classes

Whole class 1 class

## Student Perspectives

Nine classes provided feedback on a student survey, in some cases the survey was administered as a whole class oral activity, in others, as an individual student response. In many cases the students' actual wording has been retained for impact and insight.

### 1:- How good do you think you are at problem solving?



Children were asked to consider their view of themselves as mathematics problem solvers and to place themselves thus on the continuum. This graph shows the results. It captures the wide range of ability that Principals and teachers described.

### What problems do you find it easy to solve?

- Problems that use numbers less than 100
- Real life problems that use the 4 operations with tidy numbers
- Problems that you can use basic facts with and timetables less than 10x.
- Algorithms, fractions, decimals, number patterns, shape patterns, time, money problems.

### 3:- Why are these problems easy for you?

- The answers are small numbers.
- Because I am just adding on small numbers.
- Because all the problems are 20 and under.
- Because I am used to it, I am really good at it, I learnt them years ago and have used them for years, because I know them, Mathematics is just something I am good at.

- Because they are like pictures.
- We have good strategies like splitting numbers.
- Because we have practised them a lot.
- Using equipment is fun, I can use my fingers.
- Because I have been taught them well.

#### **4:- What types of problems do you find hard to solve?**

- Problems with 2 digit numbers like 83
- Adding big numbers
- Place value
- Halves, quarters, fractions, division, multiplication, subtraction
- Big numbers – over 100  $20+100$
- Large numbers  $3647-2956$ ,  $7777 + 90$
- Decimals
- Algebraic equations, numbers that are not tidy,
- Time. Money, Algebra, geometry, percentages
- Anything above 6x tables
- Expanding brackets
- Ones that I haven't been taught to understand easily.
- Most problems except very basic ones.

#### **5:- What makes these problems hard for you?**

- Adding two big numbers and having to count them all.
- Adding ones to hundreds and tens.
- I don't know it much.
- What each means – what is division, what are fractions?
- Multiple steps, keeping part of the number in your head.
- They use big words and numbers, I haven't been taught them.
- Not sure I just can't get my head around them.
- They are really complicated.
- I don't know how to do them.  
I'm not sure I just don't get it.
- The points, dashes and numbers.
- I wasn't taught how to do them.
- They just don't process in my brain.

#### **6:- What would make it easier for you to solve problems?**

- Having more group time to practise, more time to do them and learn them.
- If there were words in most equations.
- Teach it more!
- Using equipment, cubes, fingers, counters, abacus, counting on, games, mathematics, Sumdog, hundreds board, 10s frames, number lines.
- Improved vocabulary, having a tool box of strategies that work for me.
- Strategies, easy build up from simple to harder, splitting numbers, writing down the information.

- using paper, acting out the problem, visualising the problem, simple rules, simple numbers, working with a partner, doubling and halving,
- Drawing the answer rather than writing it.
- Being taught how to solve the problem, having it explained properly and privately.
- Working with someone.
- Work with the teacher.
- Finding strategies to do them.
- Ask for help when I am confused.
- Games help me learn.
- Not putting letters in algebra.
- Help!

### **I am best at solving problems**

On my own 75

With a buddy 97

In a group 54

Whole class 41

<b>Summary and Links to Research:-</b>
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The students, teachers, and Principals who have contributed to this research, have identified and described a wide range of factors that they have observed and experienced, that contribute to success in mathematical problem solving, and factors that can inhibit that success.

They have also described sound understanding and awareness of effective practice, and commitment to taking direct actions to investigate their practices. They have articulated an openness to incorporating further effective practice that is well supported by research, as being likely to promote student confidence, learning and achievement in mathematics.

The ten principles of effective pedagogy described in the publication “Effective Pedagogy in Mathematics” are well represented in the responses from teachers students and Principals. The responses show clearly that a wide range of interrelated factors contribute to effective practice, and that teachers and Principals are very aware of the factors that impact positively on student motivation, knowledge, confidence and ability in mathematics, and they consciously strive to incorporate these into their practice.

The goals of “Ka Hikitia – Managing for Success” and Ka Hikitia Accelerating Success” are evident in the outcomes of this project – namely:-

“Maori learners working with others to determine successful learning and education pathways”. “Collaborating and co-constructing”. “Ako describes a teaching and learning relationship where the educator is also learning from the student in a two way process”.

“Evidence shows that effective teaching and learning depends on the relationship between the teacher and the student, and the active engagement and motivation of the student by the teacher”.

“Effective teaching practices require learning contexts that are meaningful for the learner, accurate assessment and responsive feedback that supports further learning”.

“Maori learners gaining universal skills and knowledge needed to successfully participate in and contribute to Aotearoa New Zealand and the world”.

### **Key Themes from the schools involved in the research:-**

It is evident that children do experience difficulty in reading and interpreting mathematics problems, particularly where information is inferred. They further experience difficulty thinking logically and systematically to solve the problem – the children’s statements capture this with considerable clarity.

The explicit teaching of the key competencies of thinking – particularly logical thinking, and language symbols and texts, directly related to mathematics may support the children to fully understand and process the ideas and concepts in the problems presented.

All teachers interviewed use cooperative groupings and these are valued by the children and are very effective in supporting their learning. “Such arrangements can often provide the emotional and practical support that students need to clarify the task and identify possible ways forward”. (Effective Pedagogy in Mathematics).

However, teachers have identified that students are not achieving at their best when working alone, this is in contrast to a large number of children who indicated that they consider that they do achieve best when working alone. The children have indicated that they need more time to practise ideas and concepts. This aspect of effective pedagogy is referred to in “Effective Pedagogy in Mathematics”, under the principle “Arranging for Learning”. “Effective teachers provide students with opportunities to work both independently and collaboratively to make sense of ideas”. “Teachers should ensure that all students are given the opportunity to think and work quietly by themselves where they are not required to process the varied and sometimes conflicting perspectives of others”.

It is also clear from the responses that children are not confident in solving problems in strands other than number, and they overwhelmingly indicate that they experience difficulty where problems involve the use of large numbers. This is consistent with our experience at Pukerua Bay School. The children are very aware of their inability to manage these tasks – they have the solution – they seek to be explicitly taught – they seek to learn and they know that they don’t know.

Teachers and Principals have indicated in their responses that there is a need to support teachers in developing their professional learning and capability. This is a goal in Ka Hikitia “ Investing in people and local solutions, communities or networks of provision”. It is further described in “Effective Pedagogy in Mathematics”. “Only

with substantial content and pedagogical content knowledge, can teachers assist students in developing mathematically grounded understandings”.

“While teachers can learn a great deal by working together with a group of supportive mathematics colleagues, professional development initiatives are often a necessary catalyst for major change”.

It is clear that teachers and Principals are aware of factors that contribute to success for students, and they are committed to developing mathematical skill knowledge and understanding for every student, as well as fostering a joy and appreciation of the contribution that mathematics makes to everyday life.

“When students have opportunities to apply mathematics in everyday contexts, they learn about its value to society and its contribution to other areas of knowledge, and they come to view mathematics as part of their own histories and lives”. (Effective Pedagogy in Mathematics”)

**Conclusion:-** Children’s ability to effectively solve mathematical problems across the curriculum in rich authentic contexts and for real purposes, underpins their future success in all aspects of their lives. Supporting teachers and school leaders in actively exploring, identifying and addressing factors that are inhibiting children’s success, will ensure children’s future success and enable them to develop confidence, joy and competence in this critical learning area.

It is hoped that this report provides insights to stimulate dialogue and research within schools.

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